Collective Efficiency in Two-sided Matching

Akira Namatame[1]

Tomoko Fuku[1]   Taisei Kaizouji[2]

(1) Dept. of Computer Science,

National Defense Academy, Yokosuka, 239-8686, JAPAN,

E-mail: nama@nda.ac.jp

(2) Dvision of Social Sciences,

International Christian University,JAPAN,

E-mail: Kaizouji@nda.ac.jp

The Highlight of the Talk

Markets such as labor market, a venue for bilateral trading, require a proper matching between potential traders.

(1) Propose new matching algorithms based on the bounded rationality.

(2) Investigate the scaling-property of the matching models.

(3) Application in the trading model of the Sugarscape world.

A Model of Two-sided Matching

<The marriage model>

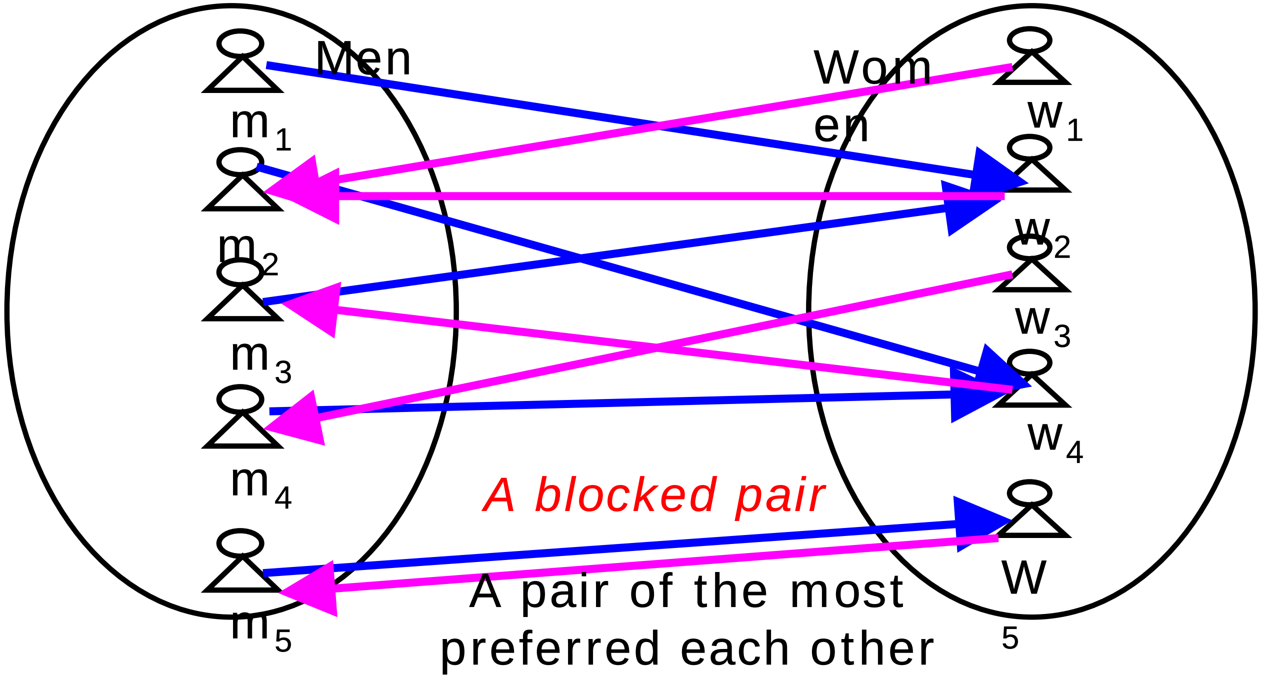
G_A(Men)= $\{m_1,..., m_n\}$, G_B(Women) = $\{w_1,..., w_p\}$

- Preference order of a man (m_i) over G_B

  $P(m_i) = w_3, w_2, \dots w_j \dots$

- Preference order of a woman (w_j) over G_A

  $P(w_j) = m_2, m_4, \dots m_i \dots$



Stable Matching

A matching τ *is blocked* by a pair of agents (m,w)

if they each prefer each other to their mates at some different matching η .

A matching τ is *STABLE* if it isn't blocked by any pair of agents.

〈Stable matching〉 **(Nash equilibrium)**

Equilibrium of **individual interest-seeking behaviors**

Deferred Acceptance Algorithm (DA)

<Step 1>

a. Each man m proposes to his 1st choice.

b. Each woman who receives more than one proposes, she "holds" the most preferred one.

<Step k>

a. Any man who was rejected at step k-1 makes a new proposal to his most preferred to acceptable mate who has not yet rejected him.

b. Each woman holds her most preferred acceptable offer to date, and rejects the rest.

Properties of DA

<Theorem> (Gale and Shapley 1968)

The matching DA produces is always stable with respect to the strict preferences.

<Theorem (Knuth)>

The best outcome for one side of the market is the worst for the other.

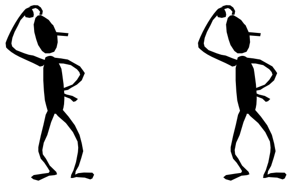
(1) Man-optimal stable matching is the worst for women.

(2) Women-optimal stable matching is the worst for men.

Scaling Properties of DA

(1) The population sizes: N *(Omero, Dzierzawa:2004)*

(2) Agent i (man) and agent j (woman) are paired.



x_i: rank order of agent j in the women

y_j: rank order of agent i in the men

$$1 \le x_i, y_j \le N$$

x(y)=1 is the best and x(y)= N is the worst

(3) Definition of energy (level of being unsatisfied)

Energy of men $E_M = \sum x_i$ Energy of men $E_F = \sum y_j$

(4) Scaling property based on random preferences

$$E_M \cong \log N \qquad E_F \cong N / \log N \qquad E_{total} = E_M \, x \, E_F \cong N$$

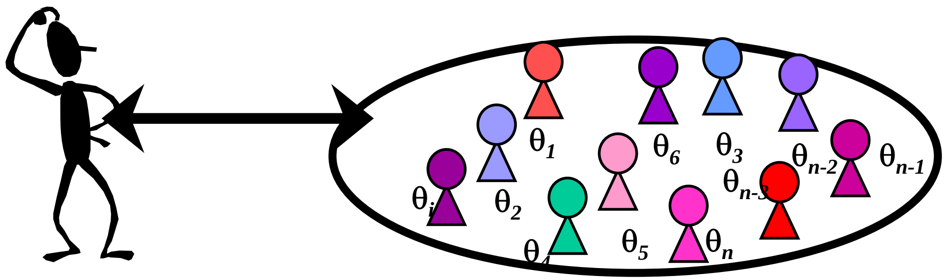
(5) The total energy of DA and the minimum energy

$$F_{total} = E_M + E_F$$

$$F_{total}^{stable} = 2\sqrt{N} \qquad F_{total}^{\min} = 1.62\sqrt{N}$$

The performance of DA is about 80% of the optimal matching

Matching Based on Bounded Rationality



(1) Model 1: **Accept if the offer is within some compromise level**

(2) Model 2: **Reject the offer if it below than some threshold**

(1) No Compromise or no threshold

Deferred Acceptance Algorithm

(2) Too much compromise (or low threshold)

Accept any offer (or hold any offer)

The optimal compromise level or threshold is set so that the social utility (collective efficiency is maximized

ASP [Acceptance with Some Patience)

<Step 1>

a. Each man proposes to his 1st choice.

b. Proposed woman accepts the proposal within the acceptance level, otherwise rejects it. (Holding is not allowed)

<Step k>

a. Each not matched woman increases the acceptable level by one

b. Any man who was rejected at step k-1 makes a new proposal to his most preferred acceptable mate who has not yet rejected him.

c. Proposed woman accepts the proposal within the acceptable level, otherwise she rejects it.

Acceptance level = Compromise level + Step number

Definitions of Utilities

N: The population size

x_i: rank order of agent j in the women

y_j: rank order of agent i in the men

x(y)=1 is the best and x(y)= N is the worst

(1) Utility of a paired individuals i and j

$$U_i = 1 - y_i / N \qquad U_j = 1 - x_i / N$$

(2) Utility of a pair of individuals i and j

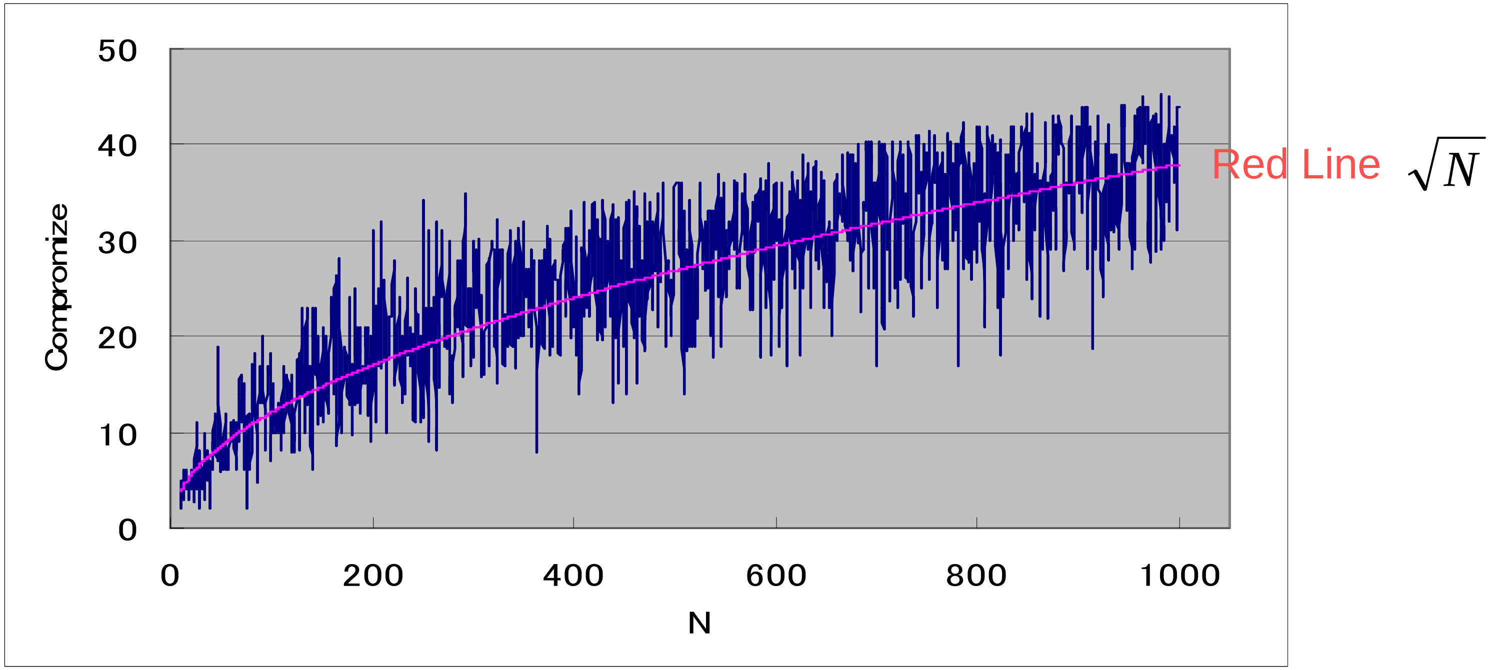
$$U(p_{ij}) = U_i \; x \, U_j$$

(3) Social Utility (Collective welfare)

$$SU = \sum_{pij} U(p_{ij})$$

The Optimal Compromise Level

Blue Line：The compromise level C to maximize social utility



<The Square Root Law of Compromise>

The optimal compromise level λ (N): $\lambda(N) = \sqrt{N}$

Advanced Deferred Algorithm (ADA)

<Step 1>

a. Each man m proposes to his 1st choice.

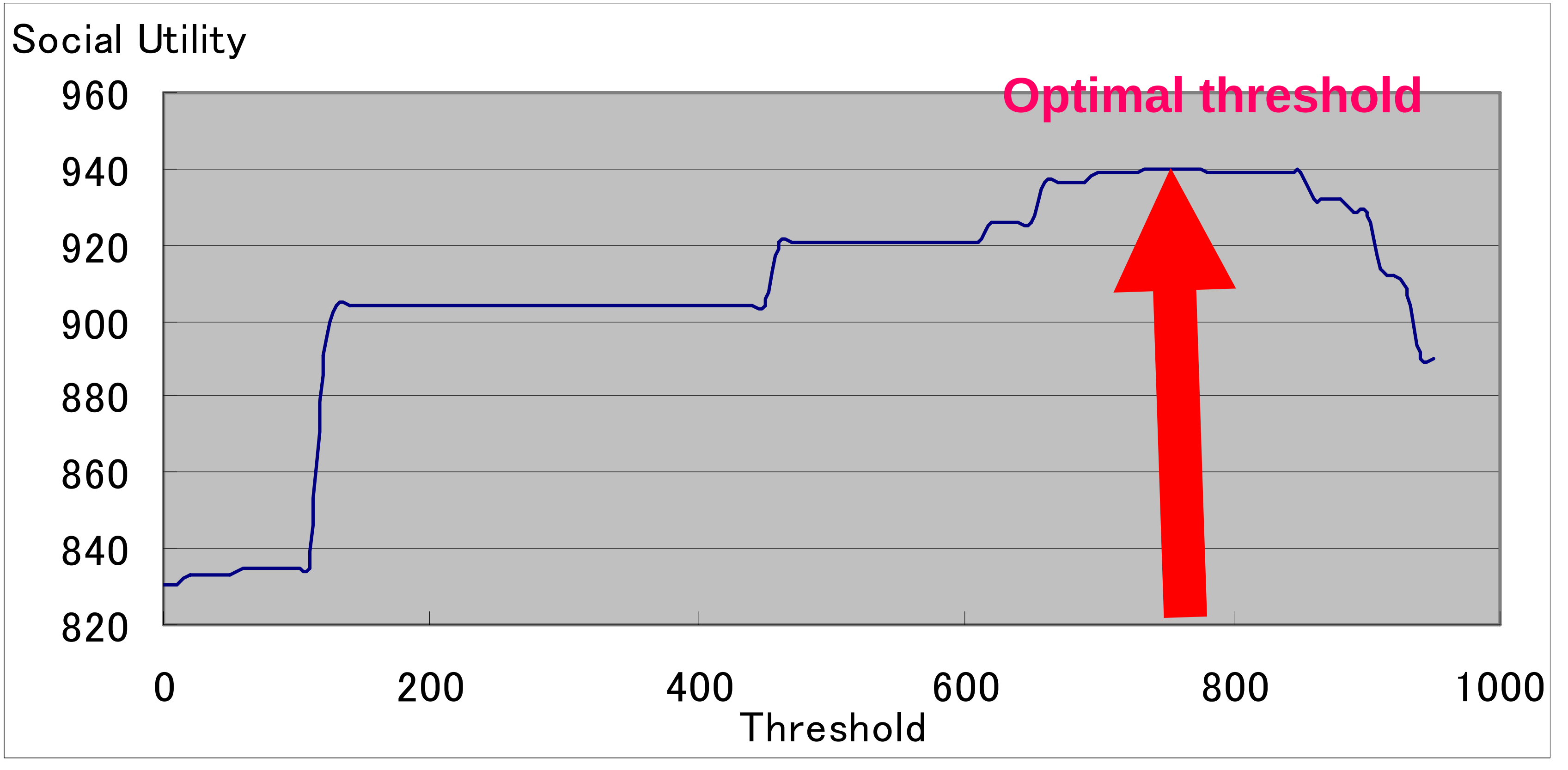
b. Each woman “rejects" the propose if it is lower than some “threshold" , otherwise “hold” the most preferred one.

<Step k>

a. Any man who was rejected at step k-1 makes a new proposal to his most preferred to acceptable mate who has not yet rejected him.

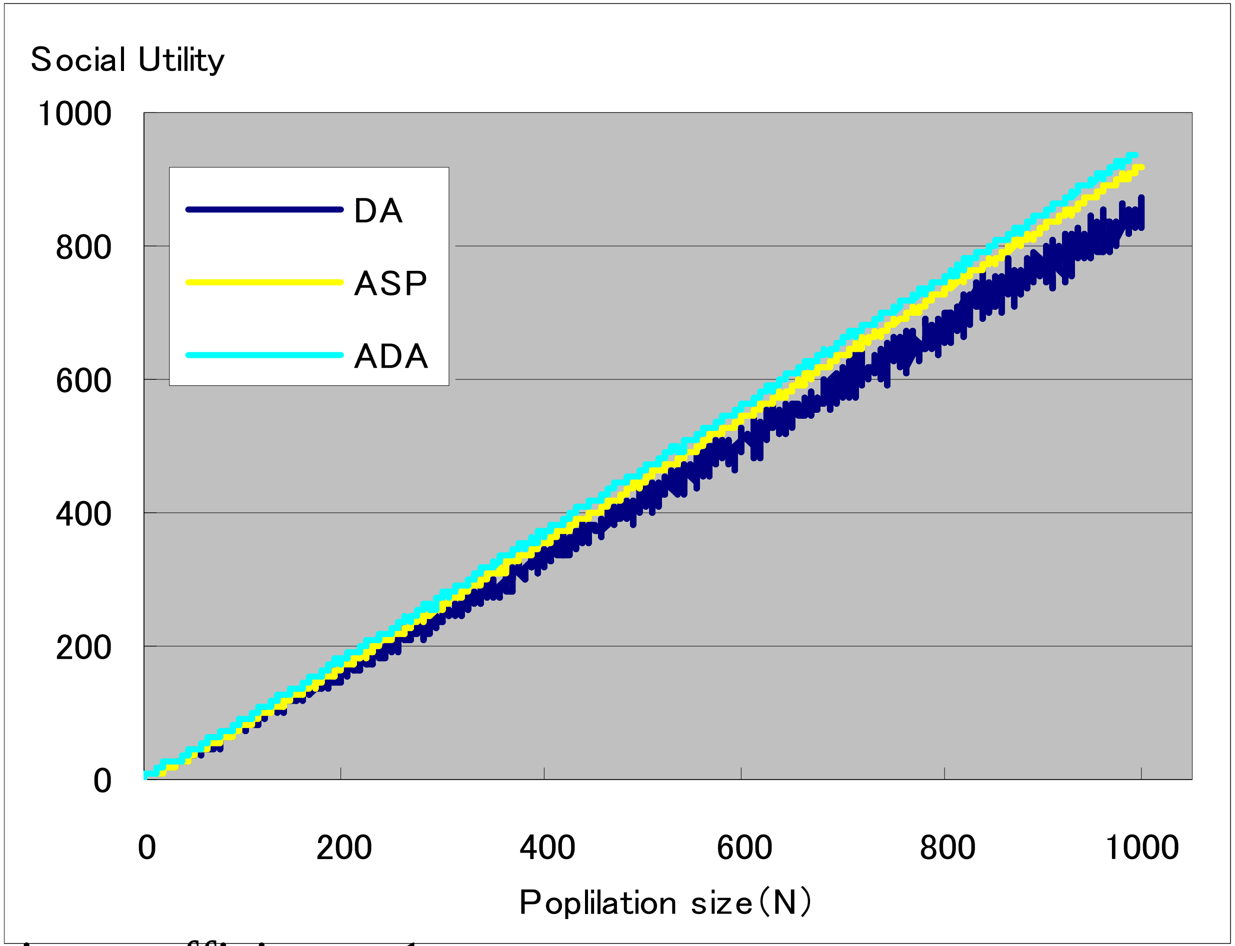
b. Each woman holds her most preferred acceptable offer to date, and rejects the rest.

The Optimal Threshold of the Cutoff



（The population of 1,000 agents）

Comparison of Collective Efficiency



The maximum efficiency: 1

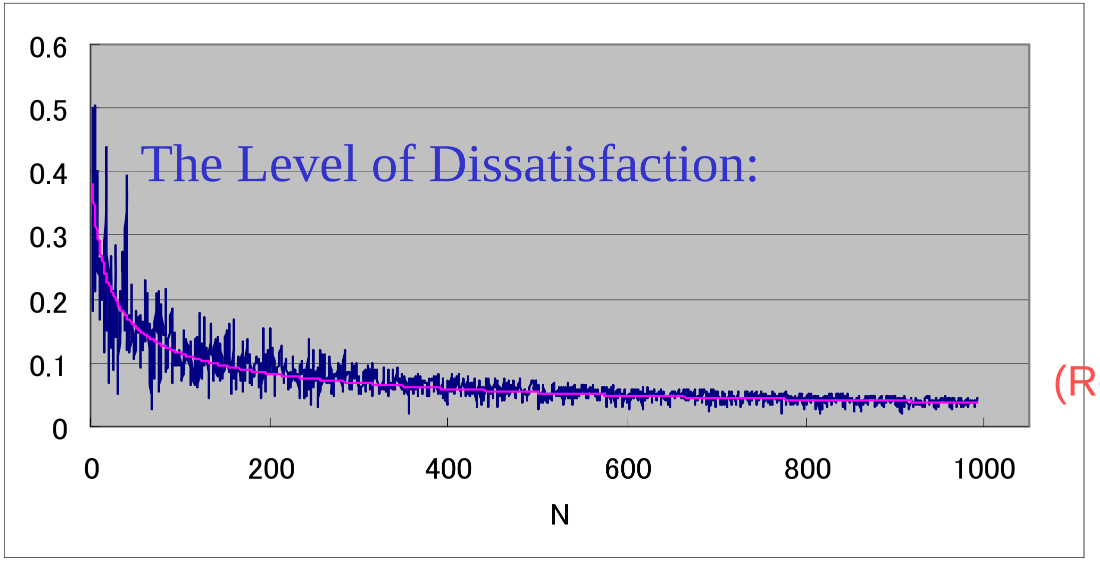
(1) Two-sided matching with compromise (ASP): 0.92

(2) Advanced Deferred Acceptance (ADA): 0.93

(3) Deferred algorithm (DA): 0.85

The Scaling Property of Collective Efficiency

$$Agent\ average\ utility \cong 1-(\sqrt{N}/N)$$

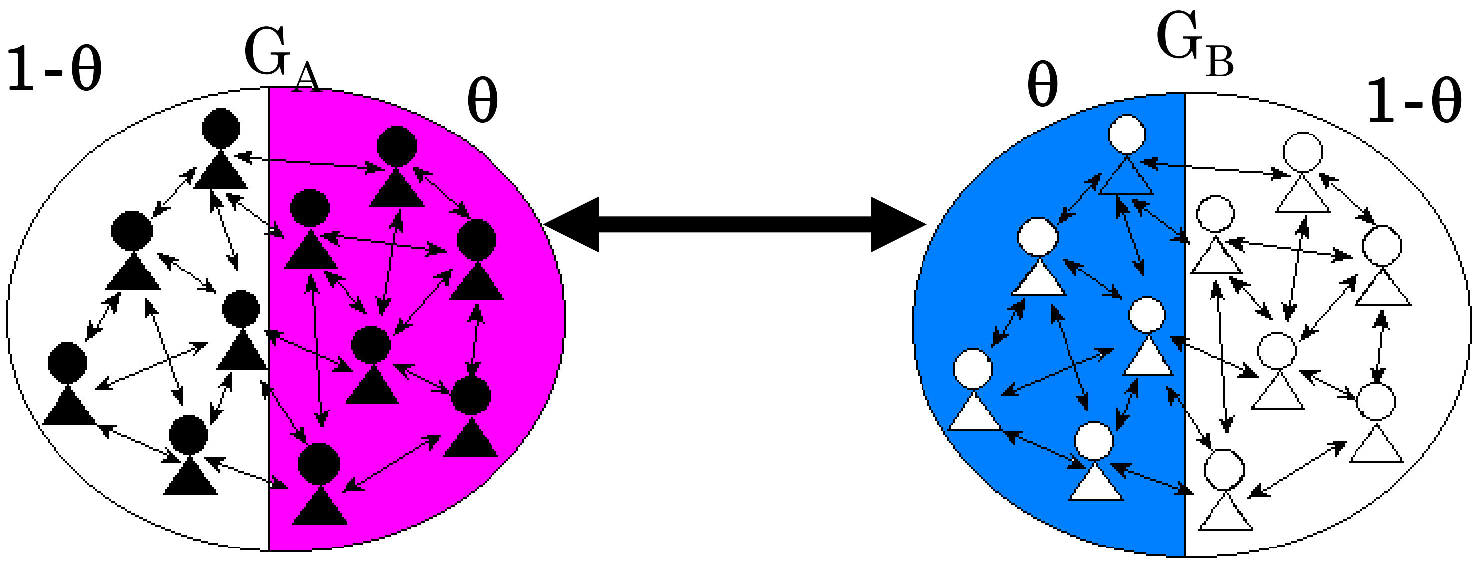


(Red Line) $\sqrt{N}/N$

Individually rational behavior results in collective rationality if the population size increase!!

Then why are there so many mismatches in our societies?

Random Preferences vs. Correlated Preferences



<Random preferences >

(1) Preferences of men and women are generated randomly.

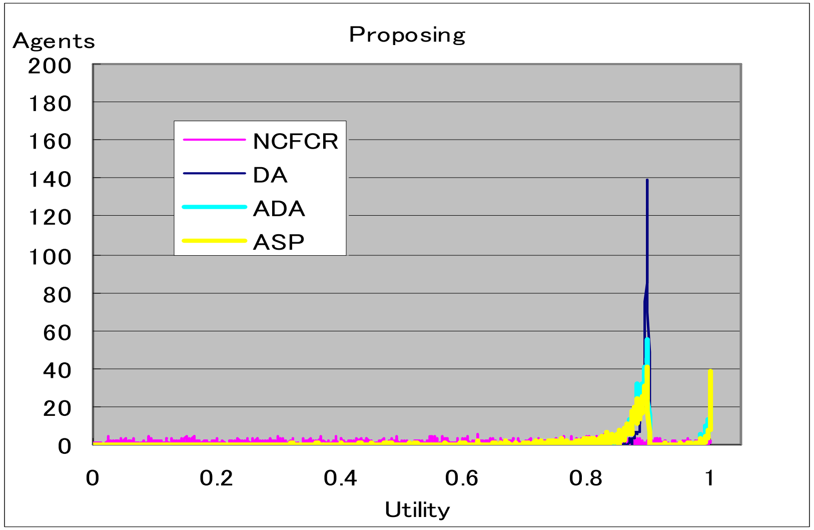
<Correlated preferences >

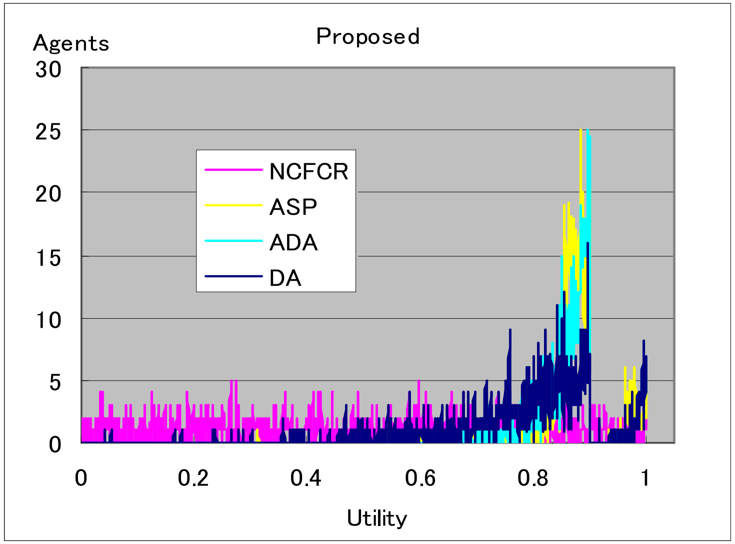
(1) There are top stars (both men and women) who are preferred by all members of the other side. (The ratio of top stars: θ)

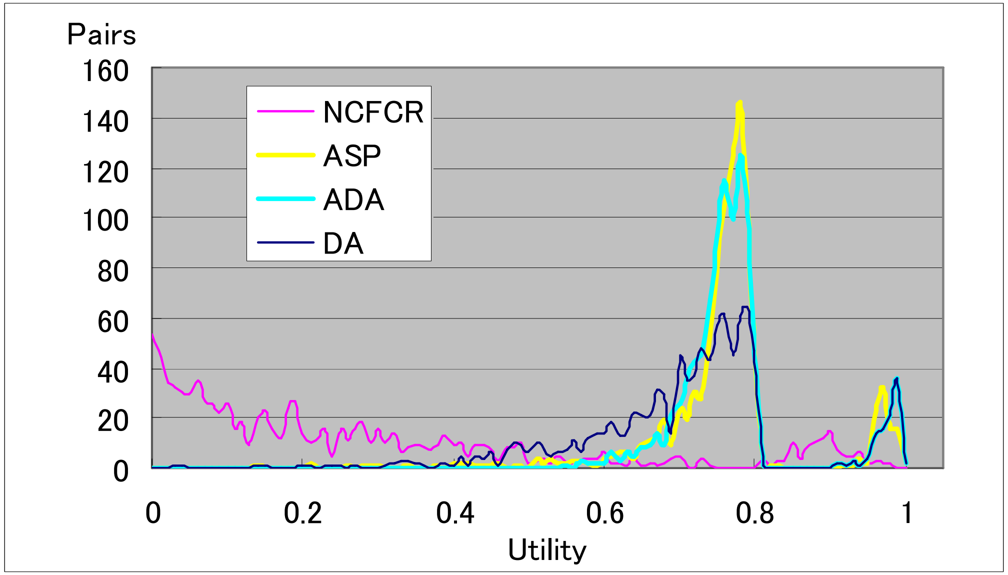
(2) The rest of the preferences (the ratio of 1- θ) are generated randomly

The Results under Correlated Preferences

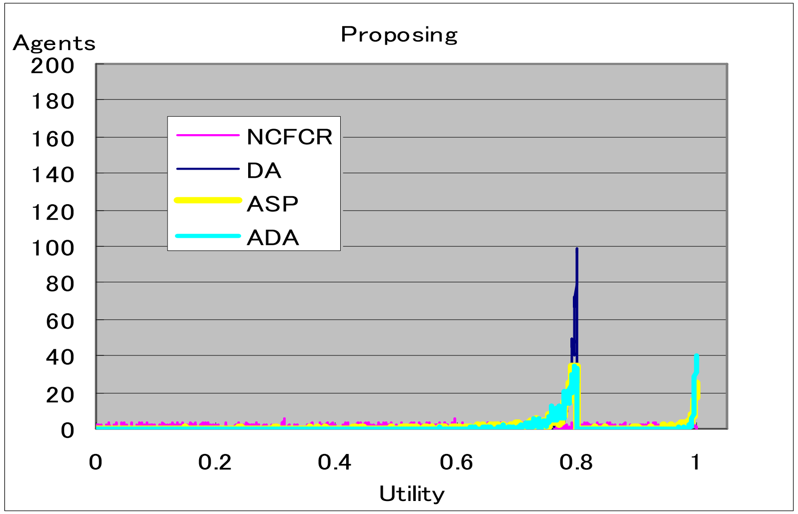
: Case 1 ($\theta = 0.1$)

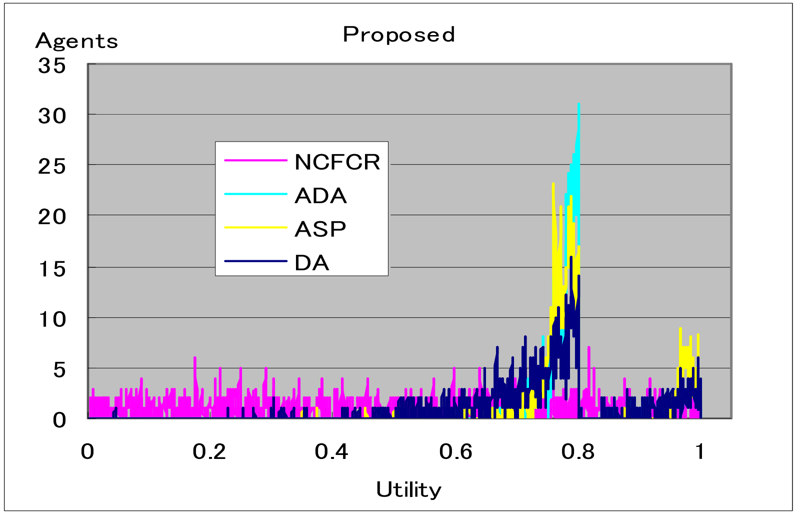


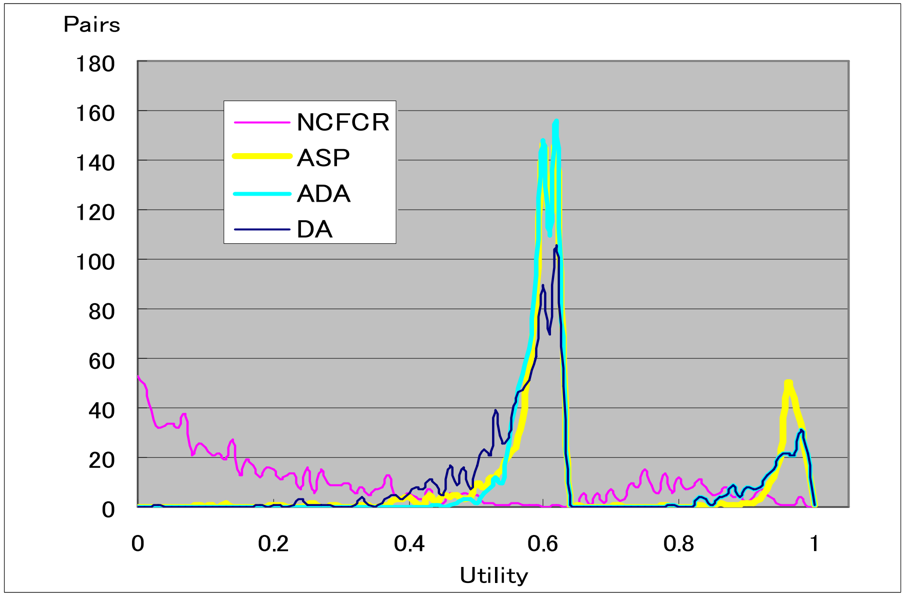




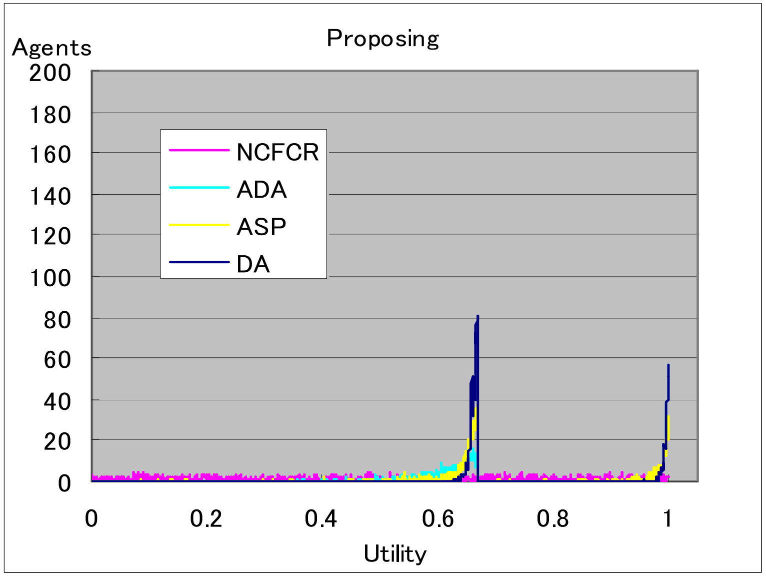
Correlated Preferences: Case 2 ($\theta = 0.2$)

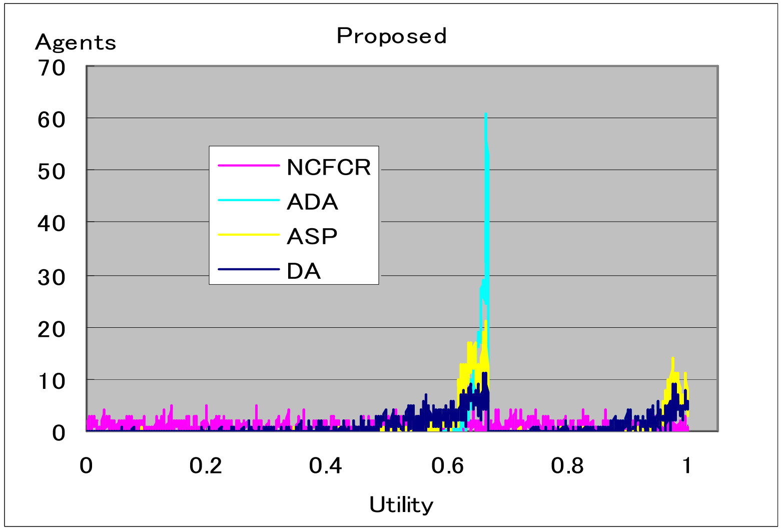


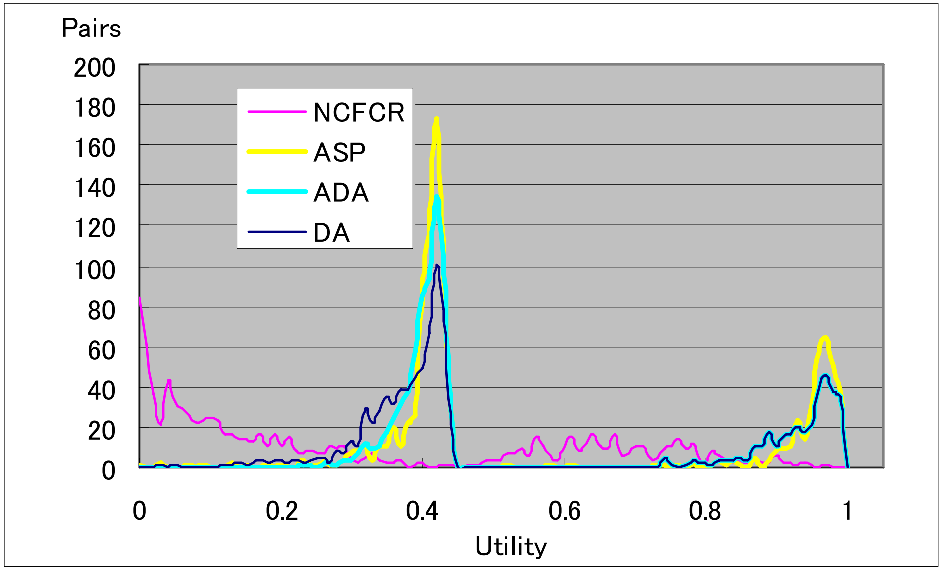




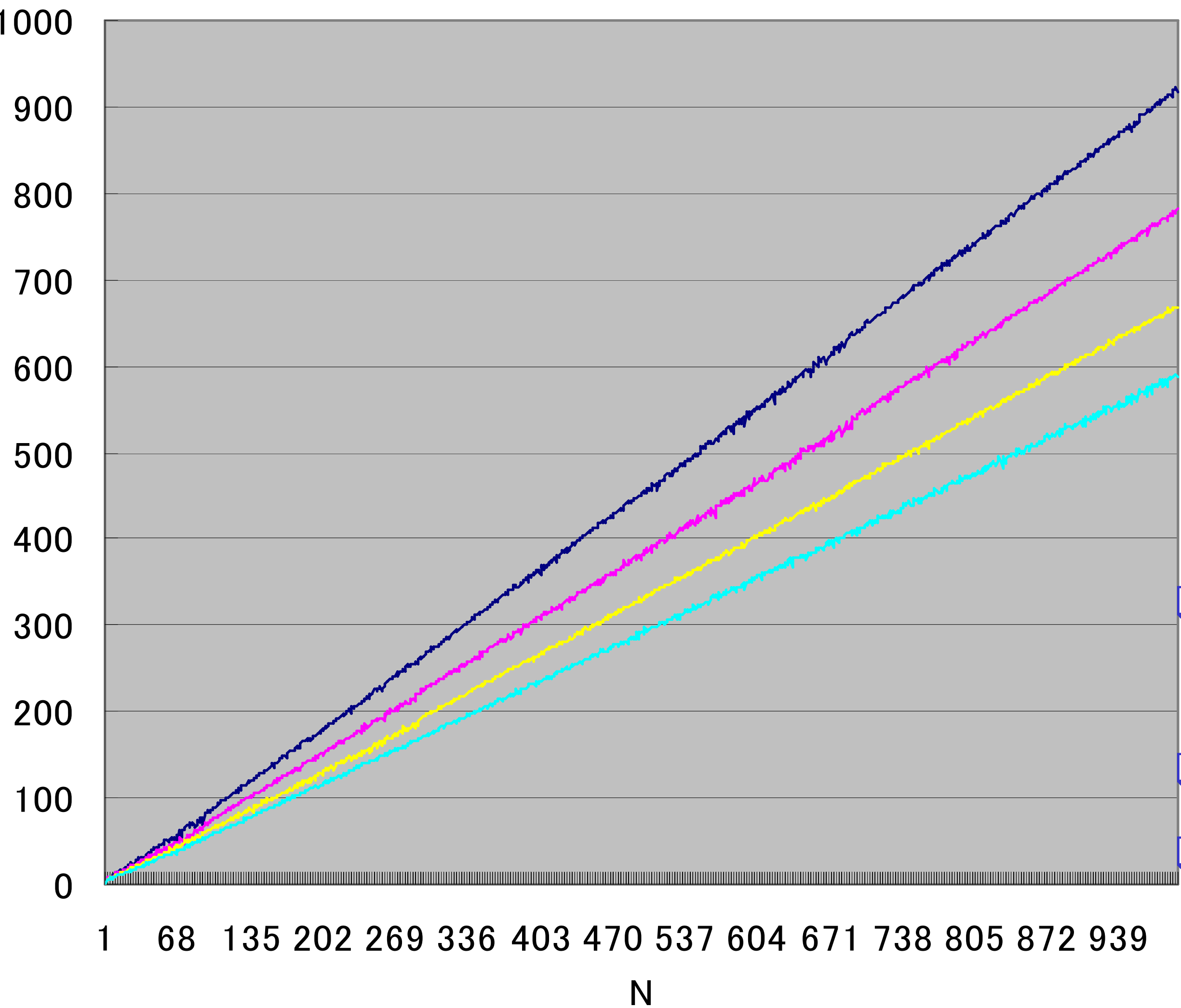
Correlated Preferences: Case 3 $(\theta = 0.33)$







Summary:
Collective Efficiency under Correlated Preferences



(The size of the population)

Correlation rate

- 0.01
- 0.1
- 0.2
- 0.33

- Random preferences

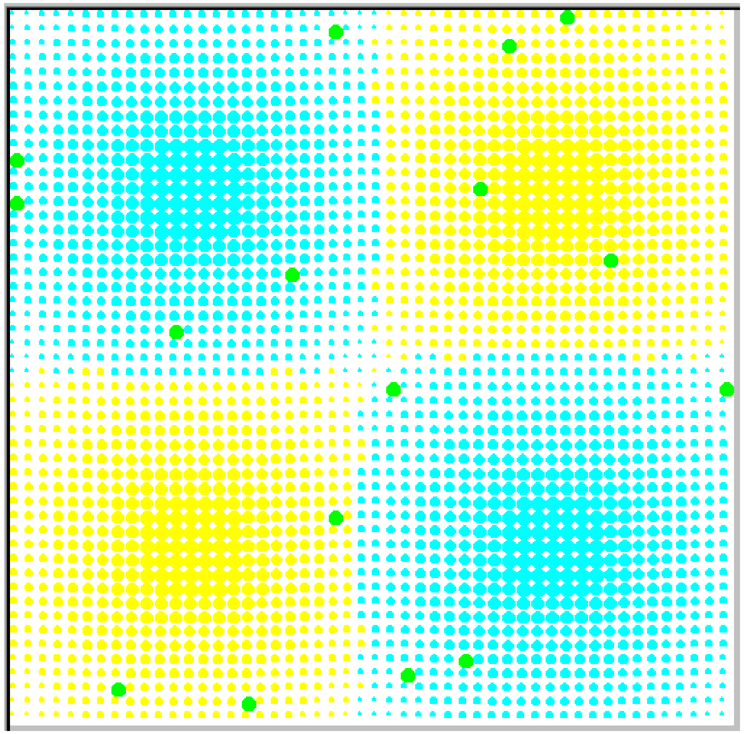
$$1-\sqrt{N}/N \cong 1$$

- Correlated preferences
- Identical preference

$$\int_0^1 (1-x)^2 dx = 1/3$$

Application of Two-sided matching in the Artificial Society

M. Epstein and R. Axtell developed the Sugarscape model for the study of new social science from bottom-up (1996)



Agent
Resource A
Resource B

They need two resources A and B to survive.

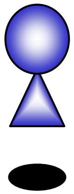
(Epstein & Axtell Model)

Epstein & R. Axtell showed that equilibrium price is achieved through local trading without market.

Bilateral Trading in the Sugarscape model

Agents trade resources to survive

Resource A *Resource* B



Metabolism for resource: m_A, m_B

Stock of resource: w_A, w_B

Utility function in terms of resources A and B :

$$W(w_A, w_B) = w_A^{m_A/m_r} w_B^{m_B/m_r} = w_A^{a} w_B^{1-a}$$

The Marginal Rate of Substitute:
(The relative price of resource A to B)

$$MRS \equiv \frac{\partial U / \partial w_A}{\partial U / \partial w_B} = \frac{w_B / m_B}{w_A / m_A}$$

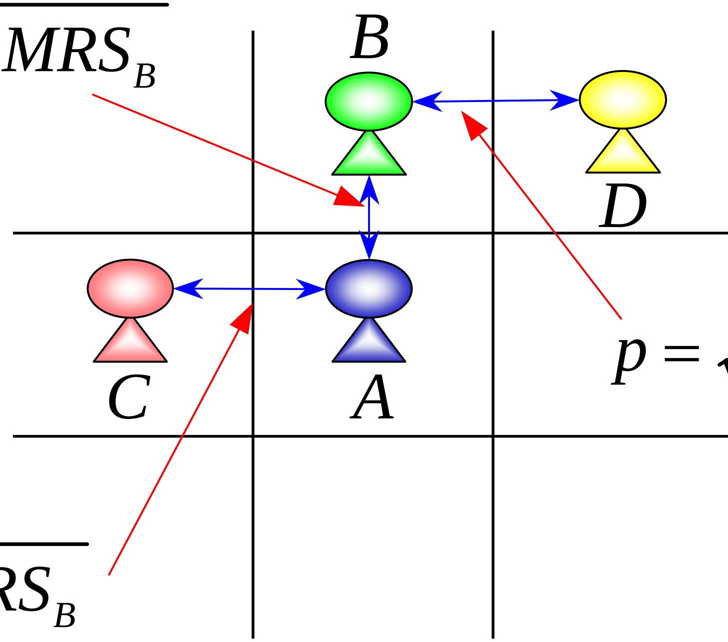
$MRS > 1 \rightarrow$ **An agent needs resource A**

$MRS < 1 \rightarrow$ **An agent needs resource B**

Bilateral Trading: Local Pricing

Model 1: Epstein & Axtell Model

$$p = \sqrt{MRS_A \times MRS_B}$$



$$p = \sqrt{MRS_A \times MRS_B}$$

$$p = \sqrt{MRS_A \times MRS_B}$$

The price is determined by the geometric average of ***MRS*** of two agents

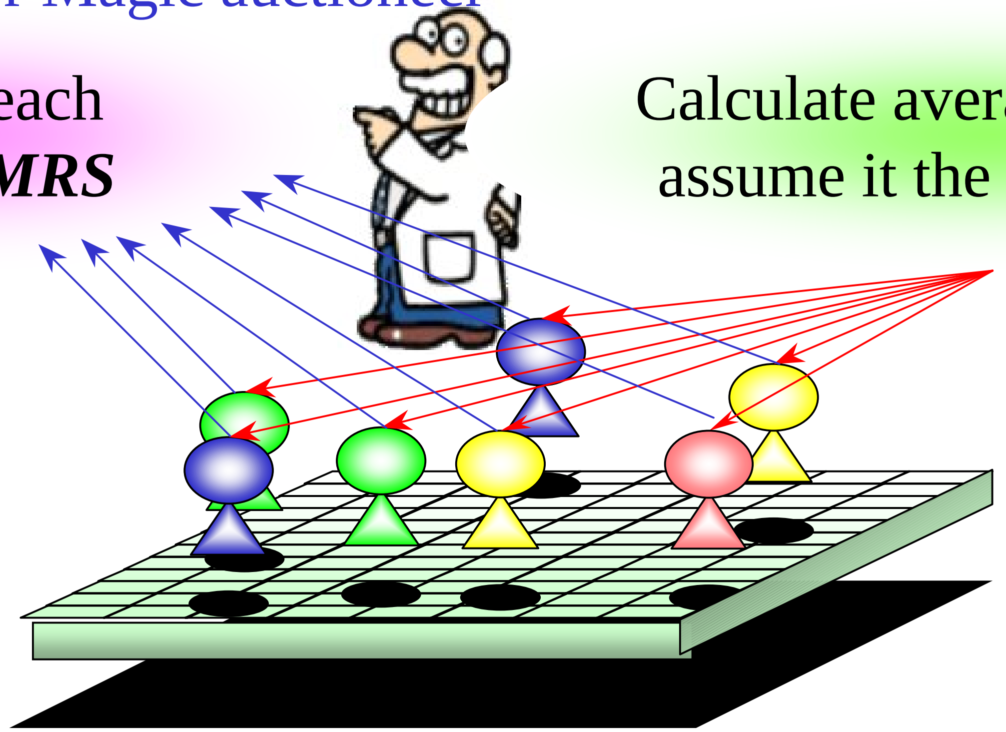
$$p = \sqrt{MRS_A \times MRS_B}$$

Global Pricing Model (Model 2)

Global market or Magic auctioneer

Collect each agent's ***MRS***

Calculate average ***MRS*** and assume it the market price



The price is decided by averaging ***MRS*** **of** all agents

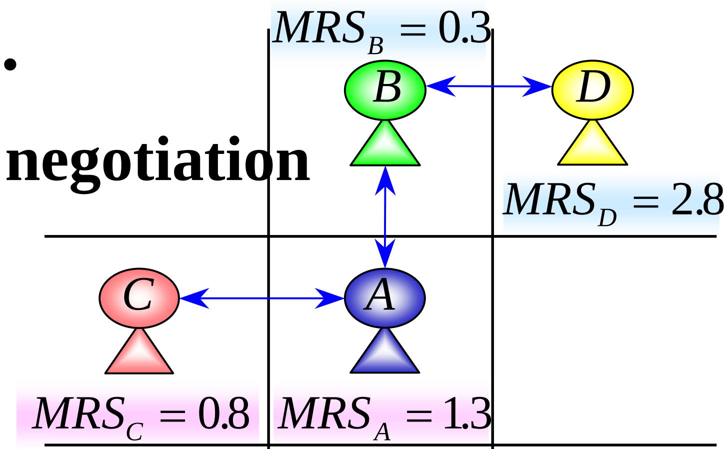
$$p = \sum_{i}^{N} MRS \,/\, N$$

Bilateral Trading by Two-sided Matching (Model 3)

(1)Trading between agent i and agent j will take place if

$MRS_i > 1$ and $MRS_j < 1$.

(2)Trading will take place under negotiation



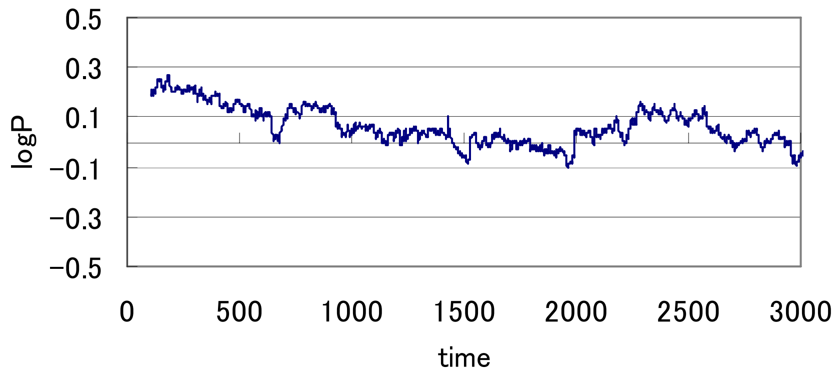
S_1: demand partner's *MR*

S_2: forgo with own *MRS*

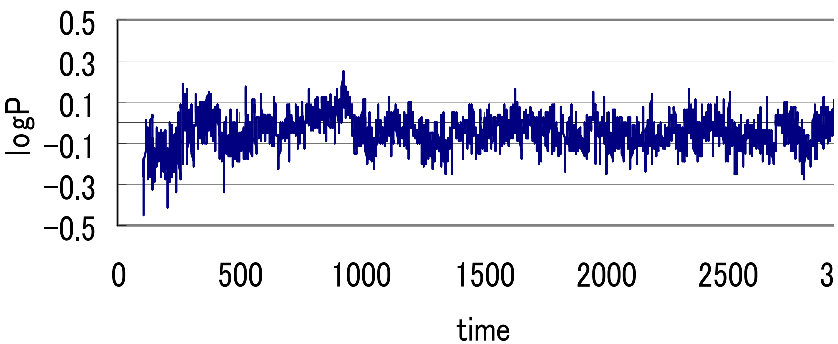
Agent *I: MRS>1*		Agent *j: MRS<1* Hawk S_1	Agent *j: MRS<1* Dove S_2
Hawk	S_1	Geometric average of ***MRS***	demand *partner's MRS*
Dove	S_2	Own's *MRS*	geometric average of ***MRS***

Comparison of Trading Prices

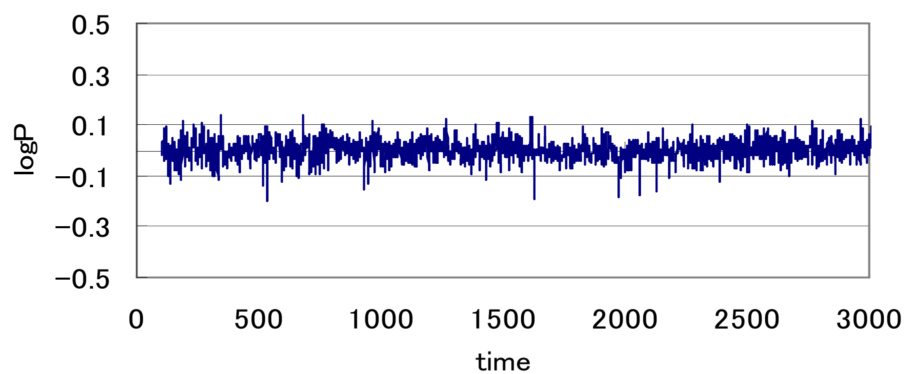
Model 2:Global pricing



Model 1: Bilateral trading

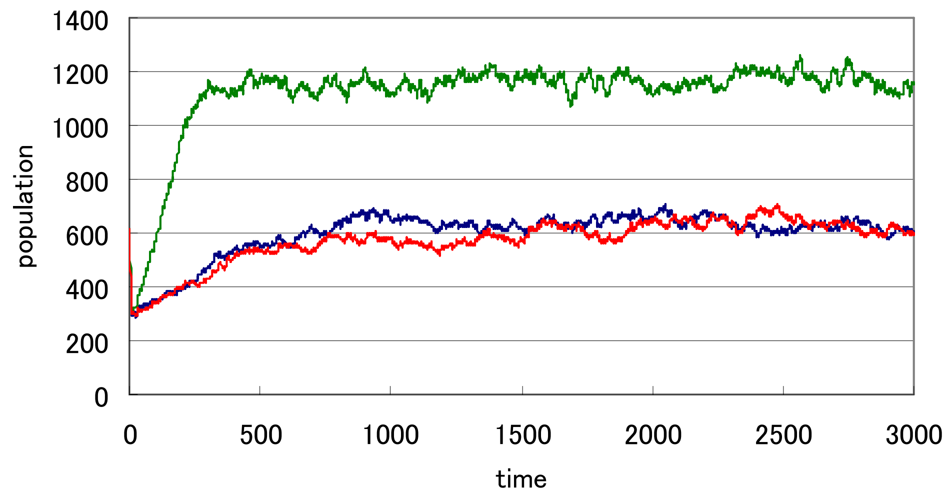


Model 3: Bilateral trading with two-sided matching



The Population Size and Average Utility (Efficiency)

<Population size>

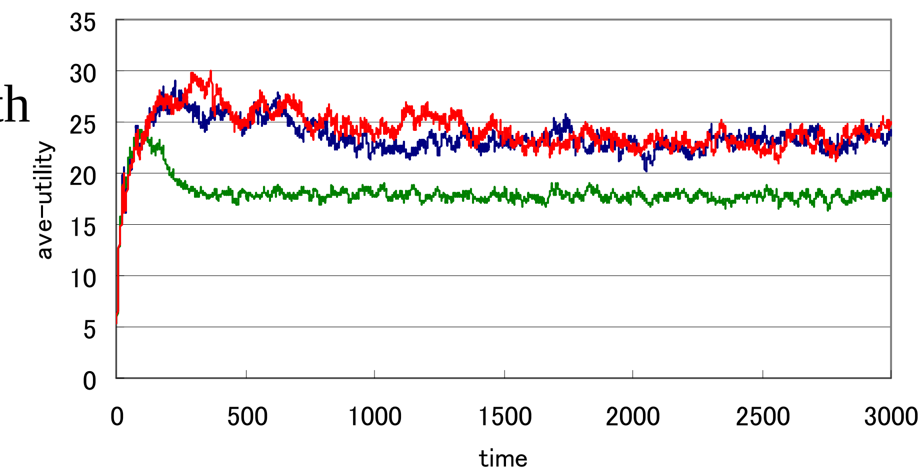


<Model 1> Bilateral trading

<Model 2> Global pricing model

<Model 3> Bilateral trading
with two-sided matching

<Utility per agent>



The performance of bilateral trading with two-sided matching is very close that of the global market model.

Conclusion

<Self-interested Hypothesis vs. Human Sociality Hypothesis>

: How far agents seek their own interest in a competitive environment?

: There are overwhelming evidences that support peoples are also motivated by concerns for fairness and reciprocity.

Compromise or cutoff behavior, which is individually irrational, improves the welfare of others.

< A small model with the precise analysis vs. A large-scale model with approximation>

We may need to explore scaling properties of large-scale socio-economic systems to convince researchers who are so reluctant to give up the the self-interested hypothesis.