Learning and the Price Dynamics of a Double-Auction Financial Market with Portfolio Traders

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Artificial Market Ingredients

We model an artificial financial market to study the interactions between the dynamics of the learning process about assets returns and the evolution of assets prices.

- Institutional market setting: double-auction market;
- Agents behavioral characteristics: liquidity constrained agents trade to reach their own target portfolio;
- We extend the model developed in A. Consiglio, V. Lacagnina, and A. Russino, *Quantitative Finance*, 2005, introducing:
  - Optimal target portfolio holdings;
  - A learning process about assets returns.

- In Consiglio et al. (2005, QF) we focused on the impact of the continuous double–auction trading mechanism:
  - making the order flow dependent on randomly assigned target portfolio allocation;
  - analyzing the role played by the agents' order–type submission strategy (market vs. limit orders). We compared two settings: one where the order–type submission strategy is exogenously specified (**ST**), another where the agents' choice of the order–type is dependent on the information revealed by the state of the book (**BA**).

Previous results

- We showed that the institutional setting of a double-auction market is sufficient to generate non–normal univariate marginal distributions of assets returns and temporal patterns resembling those observed in real markets (such as serial dependence in volatility and trading volume);
- We showed that the state of the book provides an implicit coordination device inducing agents to supply liquidity when the market needs it, therefore attenuating the frictions induced by the trading mechanism.

The market model

- N risky assets;
- M portfolio traders subject to liquidity constraints;
- A continuous double-auction market with an exogenously specified order–type submission strategy, (ST);
- Optimal portfolio choices based on agents' subjective view of the joint distribution of assets' returns updated over time using public information;

The Market model (ctd)

- Traders enter the market sequentially;
- The probability of entering the market, $P_i(E)$, is function of the total imbalance between the target and the current portfolio,

$$P_i(E) = f(\Delta_i)$$

$$\Delta_i = \sum_{j=1}^{N} \left| h^*_{ij} \, W^t_i(k) - x^t_{ij}(k) \, P^t_j(k) \right|$$

Market setting (ctd)

- At each moment in time, agents trade to adjust their portfolio according to their target allocations;
- At time step k, the number of units of the j-th asset that the i-th agent is willing to trade is given by,

$$q_{ij}^t(k) = \left\lfloor \frac{h_{ij}^*(t, t+\tau)\, W_i^t(k) - x_{ij}^t(k)\, P_j^t(k)}{P_j^t(k)} \right\rfloor$$

- For given target allocations, agents tend to act as contrarian traders: $\frac{\partial q_{ij}^t(k)}{\partial P_j^t(k)} < 0$

Key Issues

- The definition of the agent's views and the learning process to modify its views
- A model of the multivariate distribution of asset returns
- The specifications of the portfolio selection model

The learning process

We concentrate our attention on analyzing the impact on price changes of uncertainty about the univariate marginal distributions of returns:

- We allow agents to hold arbitrary marginal prior densities for the assets returns;
- We assume that agents share a common constant view of the securities association structure;
- We assume that agents correctly use a copula function to generate the N-variate returns distribution from their arbitrary set of N univariate distributions;
- Agents use the history of observed market returns to update their univariate marginals in a bayesian fashion.

The learning process (ctd)

- We use a multinomial model to represent agents' views about the univariate distributions of returns;
- For each asset, we divide the support of the returns distribution in C classes, representing the number of possible events at each trial. At each instant in time, one of the C possible events will occur;
- We model the prior marginal returns distribution of each asset as a Dirichlet with parameters $(\alpha_1, \ldots, \alpha_C)$

$$f_{ij}(\theta) \propto \Pi_{c=1}^{C} \theta_c^{\alpha_{ic}-1}$$

  where $\sum_{c=1}^{C} \alpha_c$ can be interpreted as the initial number of observations.

The learning process (ctd)

- Given β_c, the number of returns observed, for the j-th asset, in class c between two successive updating times;
- The posterior distribution of the i-th agent for the returns of asset j will be Dirichlet with parameters $((\alpha_{i1} + \beta_{i1}), \ldots, (\alpha_{iC} + \beta_{iC}))$.

Learning Speed

- We define the learning speed of the system as:

$$\text{LS} = \frac{1}{\sum_i \alpha_i};$$

- The higher is the $\sum_i \alpha_i$ (stronger views) the smaller is the effect of new information $\sum_i \beta_i$:
- As the groups of agents update their beliefs, the learning mechanism tends to homogenize the agents views. The higher is LS (weaker views) the faster the market switch from heterogenous to homogenous views.

Multivariate Distribution

- We use a Gaussian copula to model the dependence between stock returns (see Malevergne and Sornette 2003.);
- The correlation matrix is kept constant;
- Given the marginal distributions and the dependence structure, agents draw a random sample of scenario Ω to select their portfolio.

The portfolio model

- We use a prospect-type utility function to model the portfolio choice problem (Kahneman and Tversky, 1979; Benartzi and Thaler 1995; Barberis et al. 2001);
- The utility function is given in terms of deviations from a specified target, and we assume that investors are more sensitive to downside movements:

$$\begin{array}{ll} \underset{\mathbf{h}}{\text{Maximize}} & E[\tilde{U}_T] - \lambda\, E[\tilde{D}_T] \\ \text{s.t.} & \\ & \mathbf{h} \in \mathcal{H}, \end{array}$$

  where $\lambda > 0$ is the risk aversion parameter
- Consiglio et al., *Interfaces*, 2004, and *Annals of Operations Research*, 2005.

Implementation Notes and Simulation Parameters

- The artificial market is developed in C++, the optimization model is solved trough calls to GAMS or GLPK libraries.
- Main parameters are:
  - M=6000, N=3, T=2400;
  - equal initial endowments across agents;
  - $G = 12$ groups with heterogeneous priors;
  - investment horizon $H = 120, 240$ days;
  - number of scenarios for each asset $\Omega = 500$, dealing frequency 5 days;
- Sequential entry for updating: every $\frac{H}{G}$ days for one group is the end of the investment horizon.

Dependence Structure

- Positive correlation:

$$C = \begin{pmatrix} 1 & 0.55 & 0.44 \\ 0.55 & 1 & 0.23 \\ 0.44 & 0.23 & 1 \end{pmatrix}$$

- Negative correlation:

$$C = \begin{pmatrix} 1 & -0.40 & -0.23 \\ -0.40 & 1 & -0.19 \\ -0.23 & -0.19 & 1 \end{pmatrix}$$

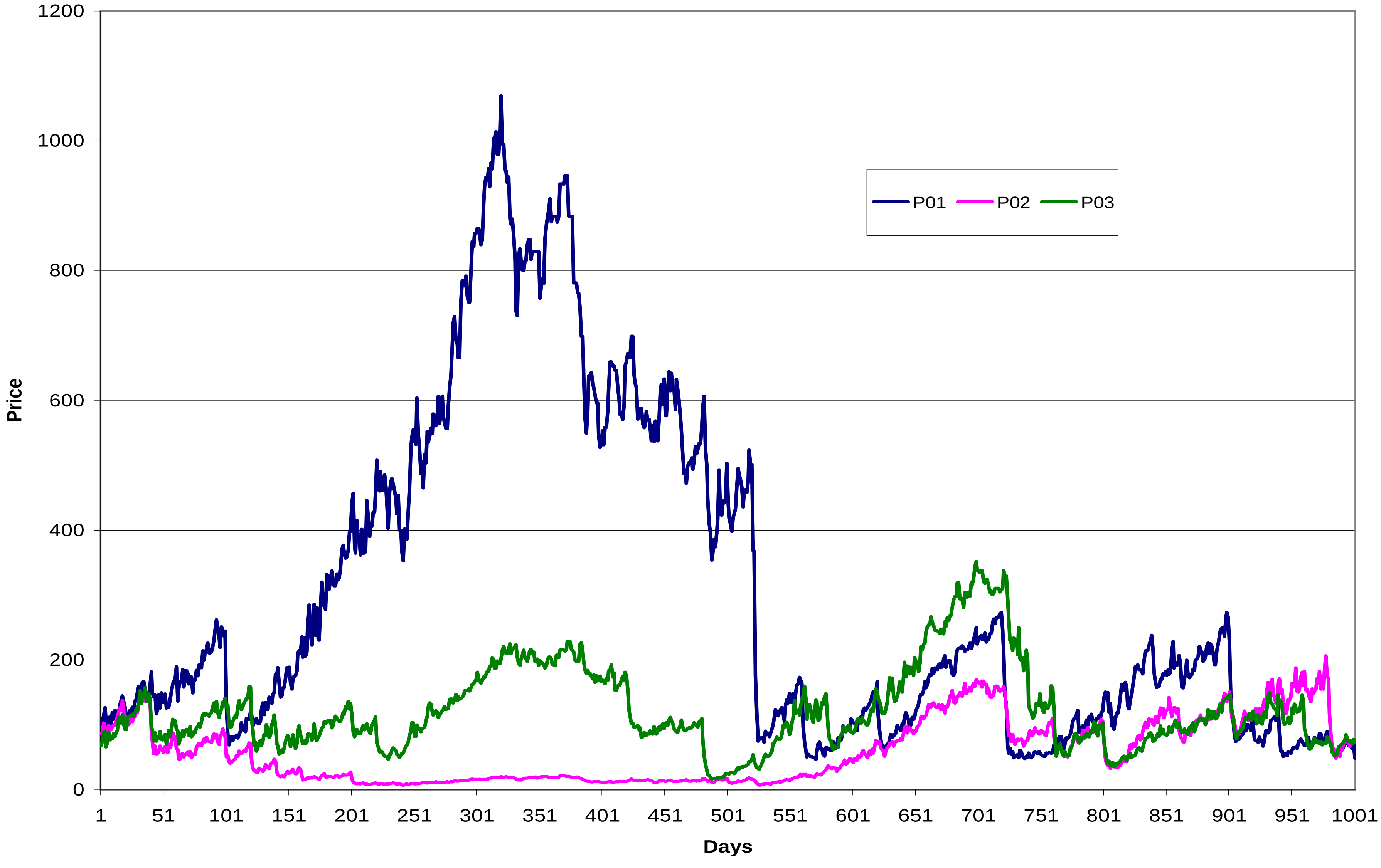
Results

- How does the price dynamics change when investors assume a positive or a negative association structure among the risky assets?

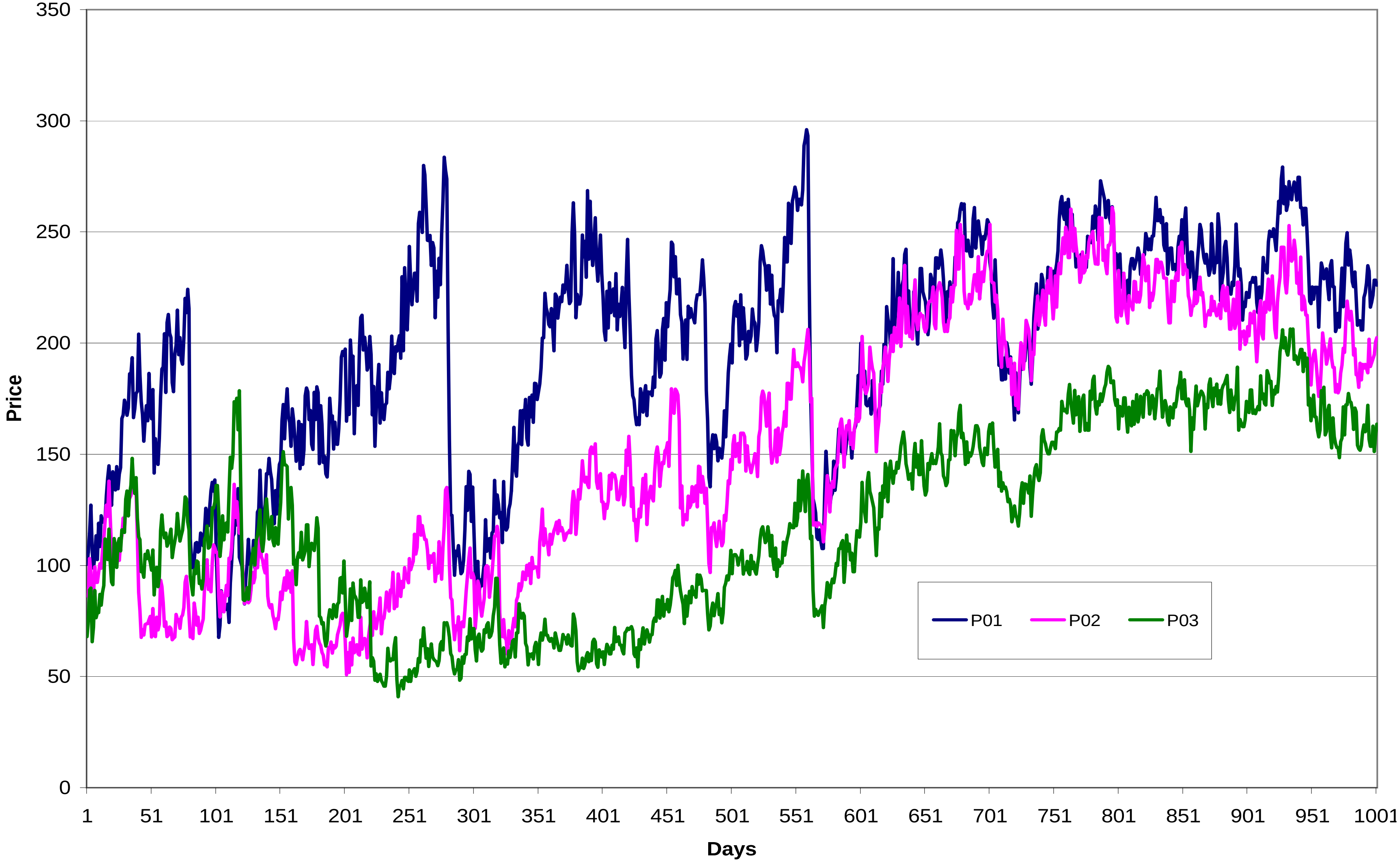
- How is the price dynamics affected by the speed of the the learning process?

- What is the role played by the length of the investment horizon?

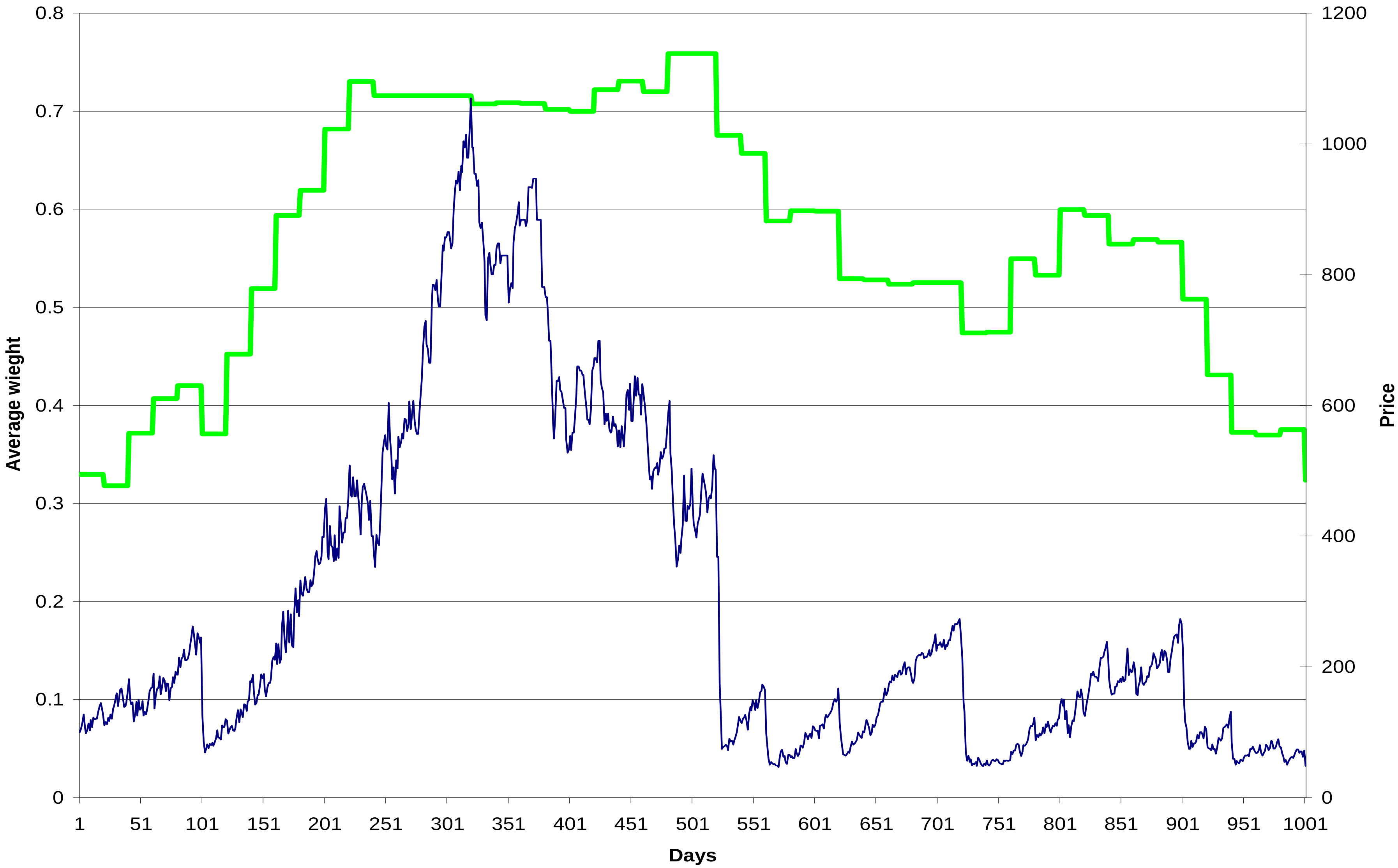
Positive Correlation



Negative Correlation



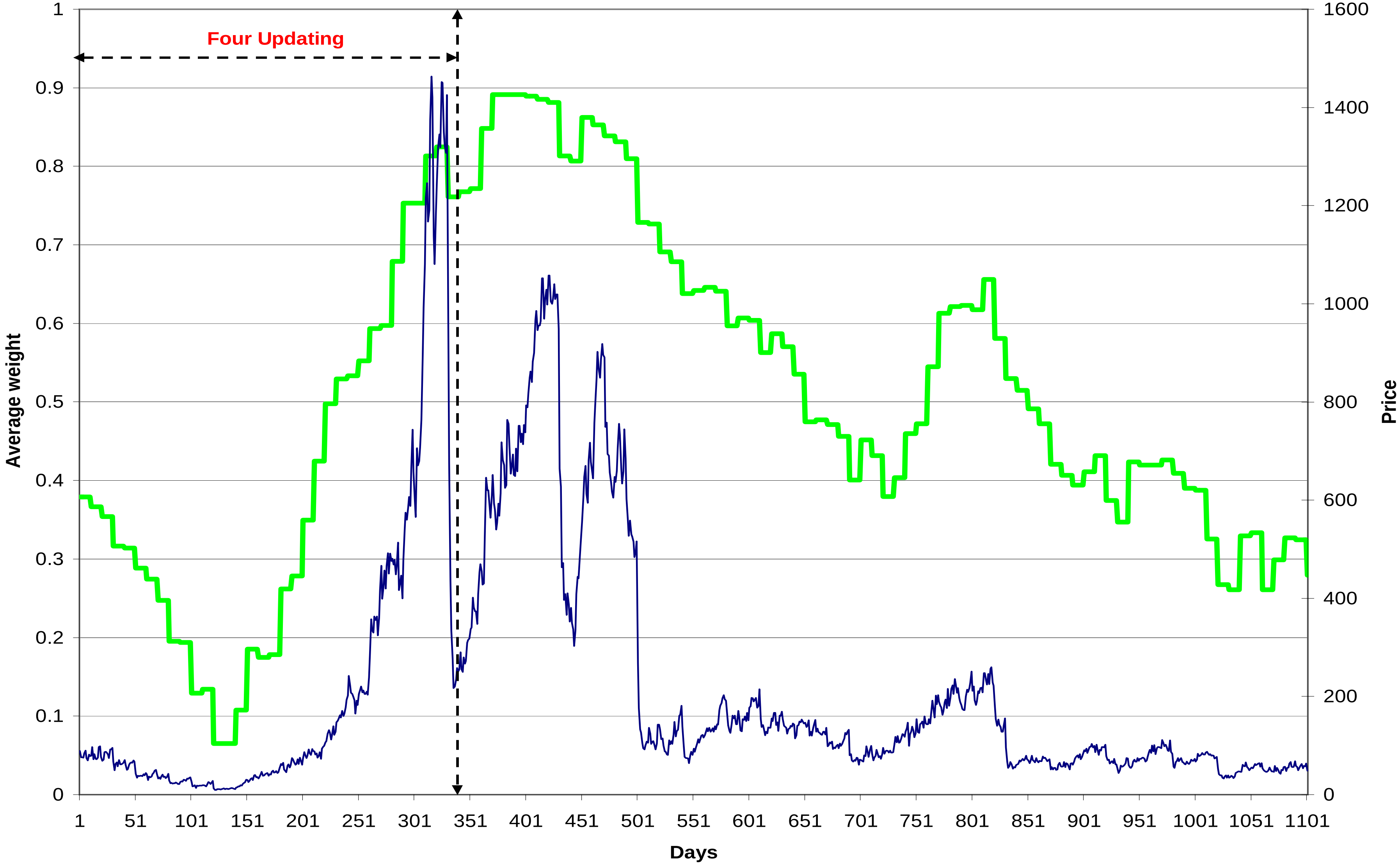
Optimal Weight and Prices: Positive Correlation



Optimal Weight and Prices: Higher Learning Speed

Optimal Weight and Prices: Different Horizon

Optimal Weight and Prices: Different Horizon



Developments

- Intraday Analysis: what drives large price fluctuations?
- Analyze cross-security effects;
- Introduce private information;
- Introduce changing risk aversion.