Firm Size Dynamics in a Cournot Computational Model

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- *Motivation*

  Explore long run firm size dynamics in a competitive and uncertain environment.

- *Research Question*

  Are simple adaptive rules capable to converge to the "optimal" equilibrium firm size?

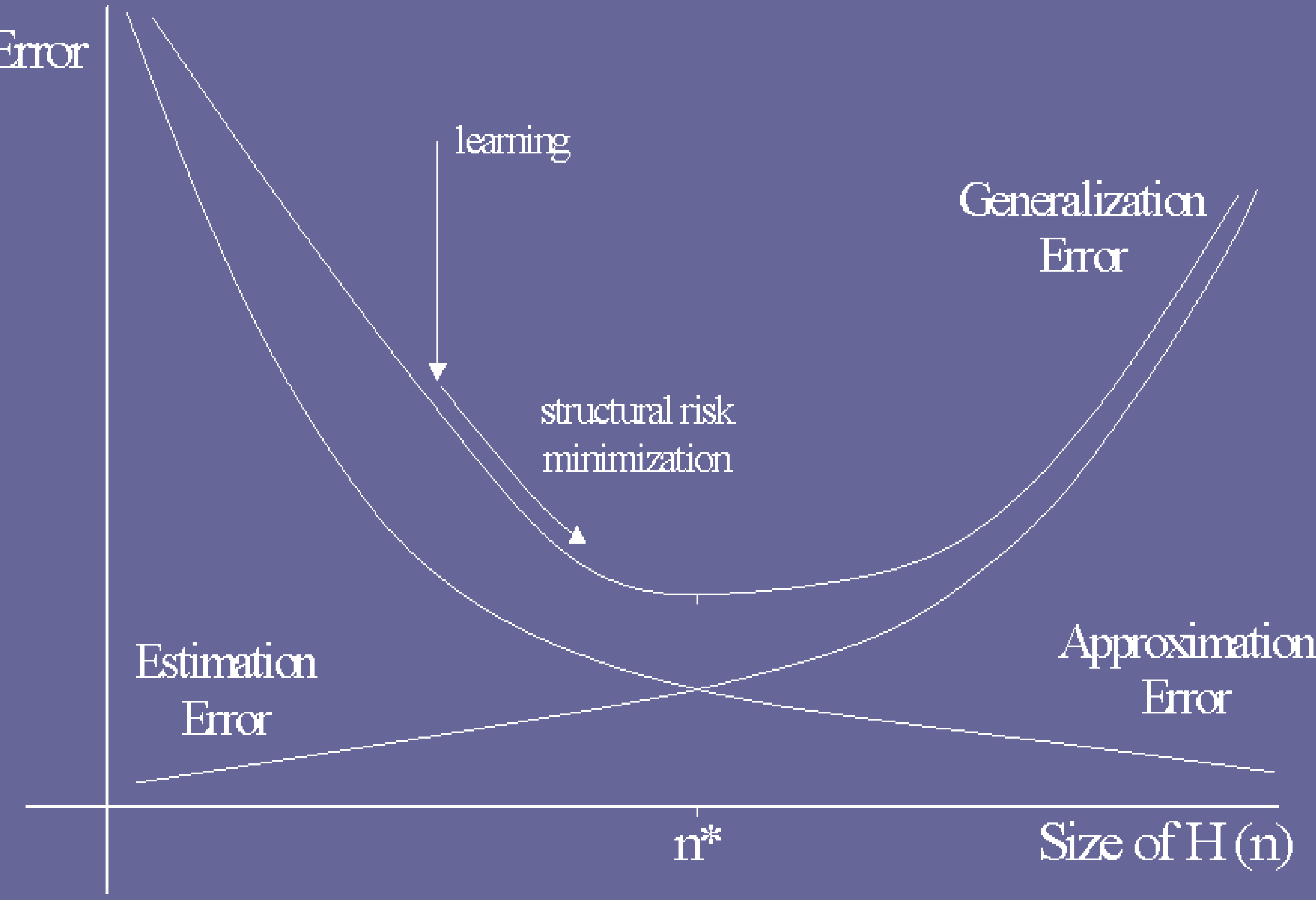
Modelling Choice

- Build on our previous work on Firms as information processing/learning machines.

- Choice of Artificial Neural Networks because:
  - ✓ Firms process information in a decentralized manner, both serially and in parallel
  - ✓ Firms learn by experience and they learn to generalize their experience to other related situations
  - ✓ The knowledge of the firm does not reside in any one agent but rather in the network.
  - ✓ Firms are capable of adapting to their environment

Central Concept of Computational Learning Theory

- Structural error/performance may be decomposed in:
  - ✓ Approximation error: Distance between best hypothesis and concept. *Increasing in Hypothesis complexity*
  - ✓ Estimation error: Capacity to select the best hypothesis (depends on sample size and time). *Decreasing in Hypothesis complexity*.
- Analogy with regression analysis


Error
learning
Generalization
Error
structural risk
minimization
Estimation
Error
Approximation
Error
n*
Size of H (n)

- ANNs: Hypothesis complexity = Number of nodes/agents.

- A general result of our previous research is the appearance of the tradeoff: Small networks quickly reach a good approximation of the concept to be learned, but large ones outperform them if only are given enough time.

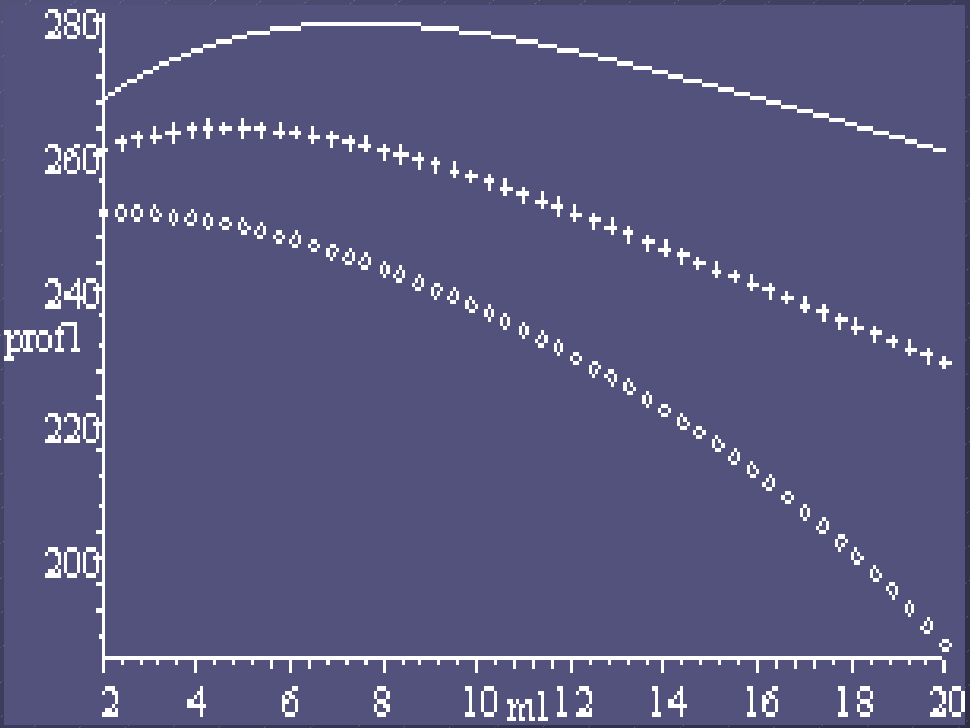
- In Cournot application, Firms have to learn the demand curve. Determinants of profit are sizes and environmental complexity:

$$\pi_i = f(m_i, m_j, n)$$

- Current paper: Learning is in the background.
- We edogenize firm size
- Via simulations and regression analysis, we derived a "reduced form" profit equation

$$\pi_1 = 271 + 5.93m_1 - 038m^2_1 + 0.007m^3_1 + 0.49m_2 - 0.3m_1m_2 - 2.2n + 0.0033n^2 + 0.007m_1m_2n - 0.016m_2n$$

- This profit is hump shaped, reflecting the tradeoff.
- The max profit is increasing in the competitor's size



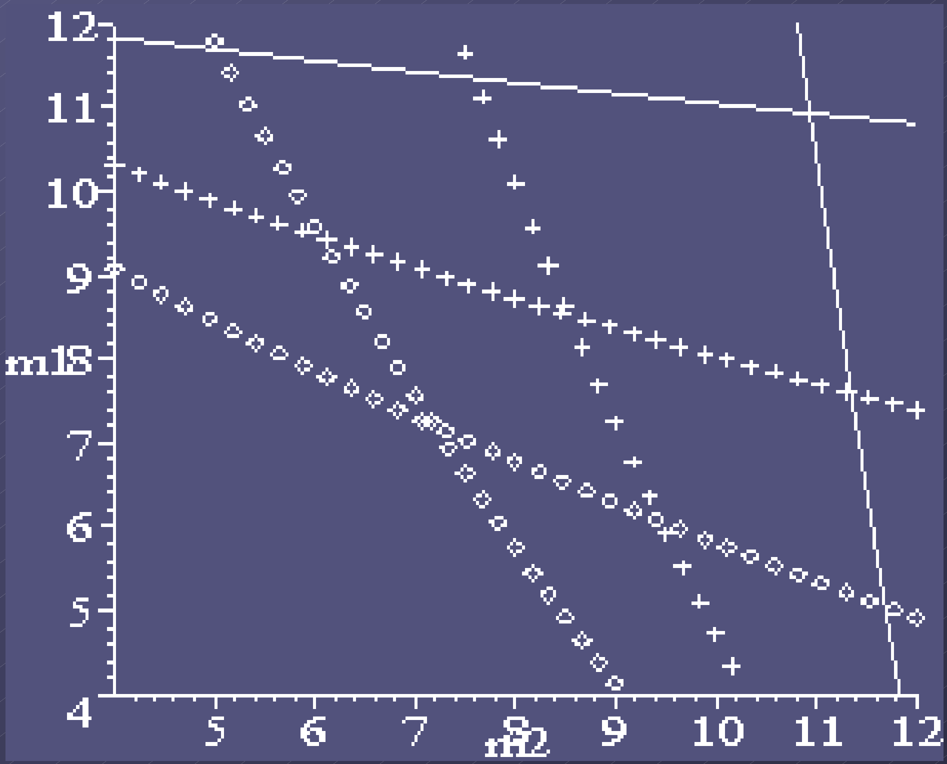
Best Response and Network Size Equilibrium

- We can derive a best response in size

$$m^{br}_1 = 16.9 \pm \sqrt{2.6 m_2 - 0.58 n m_2 + 3.9}$$

- The symmetric best response functions yield Nash equilibria in size (NSE)

- Size for these equilibria is increasing in complexity.



Adaptive Adjustment Dynamics

- Standard arguments in favor of adaptive rules: Optimization is computationally costly

- Rules of thumb may be used. General formulation:

$$m_{i,t} = m_{i,t-1} + \beta(\pi_{i,t-1} - \pi_{i,t-2}) + \alpha I_i[(m_{j,t-1} - m_{i,t-1})(\pi_{j,t-1} - \pi_{i,t-1})$$

- “Isolationist”: $\alpha=0$. Expand if profit increases

- “Imitationist”: $\beta=0$. Adjust towards the competitor if it does better.

- In the paper we analyze the two extremes

Results

- Regression analysis:
  - ✓ Run several simulations: Random draws of complexity, initial sizes, α, β, final sizes.
  - ✓ Use the dataset to run regressions and pin down the effects of different variables
- Main Results:
  - ✓ Increasing initial own size increases long run size
  - ✓ α and β have positive effects on firm size
  - ✓ Relative initial size has a positive effect. If firm 1 starts larger than firm 2, it will have a larger long run size
  - ✓ Increasing environmental complexity has a negative effect. It reduces profits and thus long run size.
  - ✓ Several interaction effects capture the non-linear relationship between the independent variables and long run firm size.

Convergence to Network Size Equilibrium

- Experiment
  - ✓ Run several simulations, random draws of complexity α and β.
  - ✓ Record runs that end in the neighborhood of NSE size.
  - ✓ Regression
- Result:
  - ✓ A very small number of runs (less than 1%) converges close to the NSE
  - ✓ Neither n nor α seem to be significant.
- Convergence to Nash is not something that can be assumed with no further discussion. The “as if” argument has to be used judiciously.

Conclusion

- Results
  - ✓ Simple dynamics are stable
  - ✓ Initial size and environmental complexity are major determinants of long run steady states
  - ✓ Simple dynamics generally do not converge to NSE

- Further research
  - ✓ More complex, though still not optimizing dynamics
  - ✓ Strategic interaction in size dynamics